

# Jet rates in the hard scattering process at finite temperature

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## Abstract

We compute the cross-section of the hadronic jets arising from the quark antiquark pair which are produced from a hard photons (of 4-momentum  $q$ ) in the plasma, predominantly consisting of thermalised quarks and gluons. The quark antiquark pair is hard and scattered off the heat bath to form jets, while the gluons being soft get thermalised in the heat bath. The infrared divergences cancel in the observable cross-section to  $\alpha_s$  order, which includes the process of emission and absorption of real gluons. Since the massless quark antiquark pair is hard the Compton scattering processes are absent in the heat bath and it renders an uncanceled collinear divergent piece in the cross-section. We regularize it by eliminating the collinear region from the phase space and write it in terms of jet parameters. The temperature dependent part of the jet cross-section is regular at large  $\frac{\sqrt{q^2}}{T}$  and vanishes when  $\frac{\sqrt{q^2}}{T} \rightarrow \infty$ . Since jets carry the thermal signature of the hot plasma the jet production rate can be used as a thermometer of the heat bath.

In this paper we consider a heat bath containing plasma of quarks, antiquarks and gluons at temperature  $T$  ( $= 1/\beta$ ). In the  $e^+e^-$  collision a hard photon (hard means it's energy and momentum are much greater than  $T$ .) is produced. Photon enters into the heat bath and produce a hard quark antiquark pair and a soft gluon (soft means it's energy and momentum are much less than  $T$ .). Since the gluon is soft it gets thermalised, however the quark antiquark pair being hard, leave the heat bath without losing much of their initial energy and form jets. We have used the real-time formalism of finite temperature field theory to calculate the imaginary part of the virtual photon polarization. We take quarks as massless. The cross-section for the real processes ( $\gamma^* \rightarrow q\bar{q}g$  and  $\gamma^*g \rightarrow q\bar{q}$ ) are infrared divergent. The divergences neatly cancel from the virtual processes ( $\gamma^* \rightarrow q\bar{q}$ ) at  $\alpha_s$  order<sup>2</sup>.

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<sup>2</sup>At temperature  $T = 0$  the KLN [1] theorem ensures that the physically measurable probabilities are infrared finite

However there still remains an uncanceled part, which is a collinear divergent piece, appearing from the real emission process ( $\gamma^* \rightarrow q\bar{q}g$ ). Since the quark antiquark pair is hard the Compton scattering processes ( $\gamma^*q \rightarrow \bar{q}g$  and  $\gamma^*\bar{q} \rightarrow qg$ ) are absent in the plasma. As a result we are left with this collinear divergent piece in the real emission process and hence also in the cross-section. We have regularised this divergent piece by eliminating the collinear region from the phase space and write it in terms of the jet parameters. Since jets carry the information about the plasma in the heat bath, the jet production rate (or cross-section) can be used as a thermometer of the heat bath<sup>3</sup> [11].

We consider the hard scattering of quark antiquark pair in the plasma. Since the gluons are very soft they get thermalised in the heat bath. In general, if both quarks and gluons are thermal the imaginary part of the photon self energy can be written as [14, 15]

$$Im\Pi_{\mu\nu}(q) = \frac{i}{2} \frac{e^{\beta q_0} - 1}{e^{\beta q_0/2}} \Pi_{\mu\nu}^{12}(q) \quad (1)$$

where  $\Pi_{\mu\nu}^{12}$  is the 12 component of the self energy. In order to obtain the imaginary part for the non-thermal quarks and the thermal gluons we will set all the sign function in the quark propagators to theta function and also set all the fermion distribution functions to zero<sup>4</sup>.

The cross-section of  $e^+e^- \rightarrow \gamma^* \rightarrow q\bar{q}g$  is given as

$$\sigma = 2Im\Pi_{\mu\nu}(q)L^{\mu\nu}. \quad (2)$$

$L^{\mu\nu}$  is the leptonic part and is given as

$$L^{\mu\nu} = \frac{e^2}{2(q^2)^3} (p_1^\mu p_2^\nu + p_1^\nu p_2^\mu - \eta_{\mu\nu} p_1 \cdot p_2), \quad (3)$$

where  $p_1$  and  $p_2$  are the 4-momenta of  $e^+$  and  $e^-$  and  $q (= p_1 + p_2)$  is the 4-momentum of the virtual photon. The imaginary part of the photon self energy,  $Im\Pi_{\mu\nu}(q)$ , contains the hadronic even when masses go to zero. Infrared divergences become severe at  $T \neq 0$  than at  $T = 0$  because the phase space integrals are weighted by the Bose-Einstein (BE) distribution function. In order to get infrared cancellations at  $T \neq 0$  KLN prescription is to include in the observable rate the emission and absorption of low energy real particles in the plasma [2-12].

<sup>3</sup>Temperature of the plasma can also be measured by studying the missing- $p_T$  spectrum of multijet events where  $p_T$  is the transverse component of the total momentum of the jet [13].

<sup>4</sup> Massless fermion and gauge field (in Feynman gauge) propagators at finite temperature read  $iS_{AB}(p) = \not{p} i\tilde{S}_{AB}(p)$  and  $iD_{AB}^{\mu\nu ab}(p) = -\eta^{\mu\nu} \delta^{ab} iD_{AB}$  ( $A, B = 1, 2$ ) respectively, where  $\eta^{\mu\nu}$  is the Minkowski metric and  $a, b$  ( $a, b = 1, \dots, N_c^2 - 1$  for  $SU(N_c)$  gauge group) are the gauge group indices.  $i\tilde{S}_{AB}$  and  $iD_{AB}$  are the  $2 \times 2$  matrices and their explicit form can be found in Ref.[15, 16].

contributions. Since the electromagnetic current is conserved we can write it as

$$Im\Pi_{\mu\nu}(q) = -\frac{1}{d-1} \left( -\eta_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) Im\Pi^\mu_\mu(q) \quad (4)$$

where  $d$  ( $= 4 - 2\epsilon$ ,  $\epsilon > 0$ ) is the space-time dimension. Since the leptons are taken to be massless, the cross-section reads

$$\sigma = -\frac{e^2}{q^4(3-2\epsilon)} Im\Pi^\mu_\mu(q), \quad (5)$$

In the following we calculate the scalar  $Im\Pi^\mu_\mu$  to obtain the jet cross-section to  $\alpha_s$  order. We shall work here in the rest frame ( $\vec{u} = 0$ ) of the heat bath.

The loop diagram corresponding to the Born amplitude of the process  $\gamma^* \rightarrow q\bar{q}$  is shown in the Fig1. The imaginary part of the trace of photon polarisation tensor read

$$\begin{aligned} Im\Pi^{(B)\mu}_\mu(q) &= (2-d)e^2 N_c (\sum_n Q_n^2) q^2 \int dR_2 \\ &= -\frac{e^2 N_c}{4\pi} q^2 \left( \frac{q^2}{4\pi\mu^2} \right)^{-\epsilon} \frac{\Gamma(2-\epsilon)}{\Gamma(2-2\epsilon)} \sum_n Q_n^2, \end{aligned} \quad (6)$$

where  $eQ_n$  is the charge of the  $n$ -th flavour quark and indices  $n$  runs over the number of quark flavours.  $dR_2$  is the differential of two particle phase space for massless quarks. Since the quarks are hard they are non-thermal and the trace of the polarisation tensor is the same as zero temperature result.

The two-loop diagrams corresponding to the real processes  $\gamma^* \rightarrow q\bar{q}g$  and  $\gamma^*g \rightarrow q\bar{q}$  are shown in the Fig2. The complete contribution of these real diagrams to the imaginary part of the trace of photon polarisation tensor is given as

$$Im\Pi^{(R)\mu}_\mu(q) = \sum_{j=a,b,c,d} Im\Pi^{(2j)\mu}_\mu(q) = \frac{e^2 g^2}{2} C(R) N_c (\sum_n Q_n^2) [J^{(0)} + J^{(em)} + J^{(abs)}], \quad (7)$$

where

$$J^{(0)} = \int dR_3 X, \quad J^{(em)} = \int dR_3 n_B(K_3) X, \quad J^{(abs)} = \int d\tilde{R}_3 n_B(K_3) X, \quad (8)$$

and

$$X(K_1, K_2, K_3) = -\frac{8(1-\epsilon)}{(K_1.K_3)(K_2.K_3)} [q^2(K_1.K_2) + (K_2.K_3)^2 + (K_1.K_3)^2 - \epsilon(K_1.K_3 + K_2.K_3)^2]. \quad (9)$$

Here  $C(R)$  is the quadratic Casimir invariant of the representation  $R$  of the quarks. For  $SU(N_c)$  gauge group  $C(R) = (N_c^2 - 1)/2N_c$ , where the quark furnishes the fundamental representations of the group.  $K_1$  and  $K_2$  are the 4-momenta of the quark and antiquark respectively and  $K_3$  is the

4-momentum of the gluon.  $dR_3$  and  $d\tilde{R}_3$  are the differential 3-body phase space for emission and absorption processes respectively for massless quarks and gluons<sup>5</sup>.

After performing the integrations the final results are

$$J^{(0)} = -\frac{q^2}{8\pi^3} \left( \frac{q^2}{4\pi\mu^2} \right)^{-\epsilon} \frac{\Gamma(2-\epsilon)}{\Gamma(2-2\epsilon)} \left[ \frac{1}{\epsilon^2} + \frac{1}{\epsilon} \left\{ \frac{3}{2} - \gamma - \ln \left( \frac{q^2}{4\pi\mu^2} \right) \right\} + \frac{1}{2} (2\gamma - 3) \ln \left( \frac{q^2}{4\pi\mu^2} \right) \right. \\ \left. + \frac{1}{2} \ln^2 \left( \frac{q^2}{4\pi\mu^2} \right) - \frac{\gamma}{2} + \frac{\gamma^2}{2} - \frac{7\pi^2}{12} + \frac{19}{4} \right], \quad (10)$$

$$J^{(em)} = \frac{q^2}{8\pi^3} \left( \frac{q^2}{4\pi\mu^2} \right)^{-2\epsilon} \frac{\Gamma(2-\epsilon)}{\Gamma(2-2\epsilon)} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left[ \frac{2}{\epsilon} \int_0^1 dx_3 x_3^{-1-2\epsilon} (1-x_3)^{1-\epsilon} n_B(\sqrt{q^2} x_3/2) \right. \\ \left. + \frac{1}{\epsilon} \frac{1-\epsilon}{1-2\epsilon} \int_0^1 dx_3 x_3^{1-2\epsilon} (1-x_3)^{-\epsilon} n_B(\sqrt{q^2} x_3/2) - \int_0^1 dx_3 x_3 n_B(\sqrt{q^2} x_3/2) \right], \quad (11)$$

$$J^{(abs)} = \frac{q^2}{8\pi^3} \left( \frac{q^2}{4\pi\mu^2} \right)^{-2\epsilon} \frac{\Gamma(2-\epsilon)}{\Gamma(2-2\epsilon)} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left[ \frac{2}{\epsilon} \int_0^\infty dx_3 x_3^{-1-2\epsilon} (1-x_3)^{1-\epsilon} n_B(\sqrt{q^2} x_3/2) \right. \\ \left. + \frac{1}{\epsilon} \frac{1-\epsilon}{1-2\epsilon} \int_0^\infty dx_3 x_3^{1-2\epsilon} (1-x_3)^{-\epsilon} n_B(\sqrt{q^2} x_3/2) - \int_0^\infty dx_3 x_3 n_B(\sqrt{q^2} x_3/2) \right]. \quad (12)$$

Virtual diagrams are shown in the Fig.3 and Fig.4. First we consider the vertex correction diagrams shown in the Fig.3. The complete contribution of the vertex correction diagrams to the imaginary part of the trace of photon polarisation tensor is given as

$$Im\Pi_\mu^{(Ve)\mu}(q) = Im\Pi_\mu^{(3a)\mu} + Im\Pi_\mu^{(3b)\mu} \\ = Im\Pi_\mu^{(Ve_0)\mu}(q) + Im\Pi_\mu^{(Ve_T)\mu}(q). \quad (13)$$

Here the  $Im\Pi_\mu^{(Ve)\mu}$  is written as the sum of the two terms: Zero temperature part and the finite temperature part. The expression for them are the following:

$$Im\Pi_\mu^{(Ve_0)\mu}(q) = \frac{e^2 g^2}{16\pi^3} C(R) N_c \left( \sum_n Q_n^2 \right) q^2 \left( \frac{q^2}{4\pi\mu^2} \right)^{-\epsilon} \frac{\Gamma(2-\epsilon)}{\Gamma(2-2\epsilon)} \left[ \frac{1}{\epsilon^2} + \frac{1}{\epsilon} \left\{ \frac{3}{2} - \gamma - \ln \left( \frac{q^2}{4\pi\mu^2} \right) \right\} \right. \\ \left. + \frac{1}{2} (2\gamma - 3) \ln \left( \frac{q^2}{4\pi\mu^2} \right) + \frac{1}{2} \ln^2 \left( \frac{q^2}{4\pi\mu^2} \right) - \frac{3}{2} \gamma + \frac{\gamma^2}{2} - \frac{\pi^2}{12} + 4 \right], \quad (14)$$

$$Im\Pi_\mu^{(Ve_T)\mu}(q) = -\frac{e^2 g^2}{16\pi^3} C(R) N_c \left( \sum_n Q_n^2 \right) q^2 \left( \frac{q^2}{4\pi\mu^2} \right)^{-2\epsilon} \frac{\Gamma(2-\epsilon)}{\Gamma(2-2\epsilon)} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \\ \left[ \frac{4}{\epsilon} \int_0^\infty dx_3 x_3^{-1-2\epsilon} n_B(\sqrt{q^2} x_3/2) + 2 \int_0^\infty dx_3 x_3 n_B(\sqrt{q^2} x_3/2) \right]. \quad (15)$$

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<sup>5</sup> 3-body phase space for emission process has the momentum conserving delta function  $\delta^{(d)}(q - K_1 - K_2 - K_3)$  in  $d$  dimension. To evaluate the phase space integrals for emission process one defines the new variables  $x_i = 2q \cdot K_i / q^2$ ,  $i = 1, 2, 3$  where  $x_1 + x_2 + x_3 = 2$  [17, 18]. For absorption process the 3-body phase space integral contains  $\delta^{(d)}(q - K_1 - K_2 + K_3)$ . To evaluate the 3-body phase space integral for absorption process one goes over to the new variable  $x_i$  ( $i = 1, 2, 3$ ) where  $x_i$  now satisfy the relation  $x_1 + x_2 - x_3 = 2$ .

Next consider the self energy correction diagrams in Fig.4. The complete contribution of the diagrams in the Fig.4 to the imaginary part of the trace of photon polarisation tensor is given as

$$Im\Pi_{\mu}^{(Se)\mu}(q) = -\frac{e^2 g^2}{16\pi^3} C(R) N_c (\sum_n Q_n^2) q^2 \left( \frac{q^2}{4\pi\mu^2} \right)^{-2\epsilon} \frac{\Gamma^2(2-\epsilon)}{\Gamma^2(2-2\epsilon)} \frac{2(1-\epsilon)}{\epsilon} \times \int_0^\infty dx_3 x_3^{1-2\epsilon} n_B(\sqrt{q^2} x_3/2). \quad (16)$$

To obtain the total cross-section for the processes  $e^+e^- \rightarrow \gamma^* \rightarrow q\bar{q}g$  and  $e^+e^- \rightarrow \gamma^*g \rightarrow q\bar{q}$  we have to sum the real and the virtual process contributions. Therefore using the formula in eqn.(5) the total cross-section to  $\alpha_s$  order reads

$$\begin{aligned} \sigma &= -\frac{e^2}{q^4(3-2\epsilon)} \left[ Im\Pi_{\mu}^{(B)\mu}(q) + Im\Pi_{\mu}^{(R)\mu}(q) + Im\Pi_{\mu}^{(Ve)\mu}(q) + Im\Pi_{\mu}^{(Se)\mu}(q) \right] \\ &= \sigma_B \left[ 1 + \frac{\alpha_s}{\pi} C(R) \left\{ \gamma - \frac{\pi^2}{2} + \frac{3}{4} + \int_0^1 dx_3 x_3 n_B(\sqrt{q^2} x_3/2) + 3 \int_0^\infty dx_3 x_3 n_B(\sqrt{q^2} x_3/2) \right. \right. \\ &\quad \left. \left. + F \right\} \right], \end{aligned} \quad (17)$$

where  $\alpha = \frac{e^2}{4\pi}$ ,  $\alpha_s = \frac{g^2}{4\pi}$  and  $\sigma_B$  is called the Born cross-section and is given as

$$\sigma_B = \frac{4\pi\alpha^2}{q^2} N_c (\sum_n Q_n^2) \left( \frac{q^2}{4\pi\mu^2} \right)^{-\epsilon} \frac{\Gamma(2-\epsilon)}{(3-2\epsilon)\Gamma(2-2\epsilon)}. \quad (18)$$

Here  $F$  contains the finite temperature divergent contributions from the real and the virtual diagrams. Its complete expression is

$$\begin{aligned} F &= \left( \frac{q^2}{4\pi\mu^2} \right)^{-\epsilon} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left[ \frac{4}{\epsilon} \int_0^\infty dx_3 x_3^{-1-2\epsilon} n_B(\sqrt{q^2} x_3/2) \right. \\ &\quad + \frac{2}{\epsilon} \int_0^\infty dx_3 x_3^{1-2\epsilon} n_B(\sqrt{q^2} x_3/2) \\ &\quad - \frac{2}{\epsilon} \int_0^1 dx_3 x_3^{-1-2\epsilon} (1-x_3)^{1-\epsilon} n_B(\sqrt{q^2} x_3/2) \\ &\quad - \frac{2}{\epsilon} \int_0^\infty dx_3 x_3^{-1-2\epsilon} (1+x_3)^{1-\epsilon} n_B(\sqrt{q^2} x_3/2) \\ &\quad - \frac{1-\epsilon}{\epsilon(1-2\epsilon)} \int_0^1 dx_3 x_3^{1-2\epsilon} (1-x_3)^{-\epsilon} n_B(\sqrt{q^2} x_3/2) \\ &\quad \left. - \frac{1-\epsilon}{\epsilon(1-2\epsilon)} \int_0^\infty dx_3 x_3^{1-2\epsilon} (1+x_3)^{-\epsilon} n_B(\sqrt{q^2} x_3/2) \right]. \end{aligned} \quad (19)$$

After the cancellation of infrared divergences between the virtual and real processes finite part of  $F$  reads

$$\begin{aligned} F &= \frac{1}{\epsilon} \int_1^\infty \frac{dx_3}{x_3} ((x_3-1)^2 + 1) n_B(\sqrt{q^2} x_3/2) - 2 \int_1^\infty \frac{dx_3}{x_3} ((x_3-1)^2 + 1) \ln x_3 n_B(\sqrt{q^2} x_3/2) \\ &\quad + \int_0^1 \frac{dx_3}{x_3} (1 + (1-x_3)^2) \ln(1-x_3) n_B(\sqrt{q^2} x_3/2) \end{aligned}$$

$$\begin{aligned}
& + \int_0^\infty \frac{dx_3}{x_3} ((x_3 + 1)^2 + 1) \ln(1 + x_3) n_B(\sqrt{q^2} x_3/2) \\
& - \left( \gamma + \ln\left(\frac{q^2}{4\pi\mu^2}\right) \right) \int_1^\infty \frac{dx_3}{x_3} ((x_3 - 1)^2 + 1) n_B(\sqrt{q^2} x_3/2) \\
& - \int_0^1 dx_3 x_3 n_B(\sqrt{q^2} x_3/2) - \int_0^\infty dx_3 x_3 n_B(\sqrt{q^2} x_3/2).
\end{aligned} \tag{20}$$

All the terms in eqn.(20) are finite except for the first one. The integral in the first term is finite, however due to the  $\frac{1}{\epsilon}$  factor before it, it becomes divergent when  $\epsilon \rightarrow 0$ . This divergence appears when the gluon momentum is parallel to quark or antiquark<sup>6</sup>. The natural way of regularizing this divergence is to eliminate the potentially dangerous collinear integral region from the phase space. One can eliminate the regions around  $\theta = 0$  ( $0 \leq \theta \leq \theta_m$ ) and  $\theta = \pi$  ( $\pi - \theta_m \leq \theta \leq \pi$ ) in the angular integral. Then the infrared divergence would have manifested itself as singularity in  $\theta_m$  for its vanishing limit. If the resolving power of the detector is less than  $\theta_m$  it can not be able to detect the soft gluons as separate entity from the quarks. So we can relate  $\theta_m$  to the maximum power of the detector to detect the two particles separately, which are coming towards it with a small angular separation between themselves. In order to express this  $\frac{1}{\epsilon}$  factor in terms of  $\theta_m$  we look back into the evaluation of  $J^{(em)}$  and the relation is

$$\frac{1}{\epsilon} \leftrightarrow 2 \ln(\theta_m/2). \tag{21}$$

Then the cross-section reads

$$\sigma = \sigma^{(0)} + \sigma^{(T)}, \tag{22}$$

where  $\sigma^{(0)}$  and  $\sigma^{(T)}$  are the zero temperature and finite temperature piece of the total cross-section respectively. The expression for them are

$$\sigma^{(0)} = \sigma_B \left[ 1 + \frac{\alpha_s}{\pi} C(R) \left\{ \gamma - \frac{\pi^2}{2} + \frac{3}{4} \right\} \right], \tag{23}$$

$$\begin{aligned}
\sigma^{(T)} = & \sigma_B \frac{\alpha_s}{\pi} C(R) \left[ \frac{4}{3} \pi^2 \frac{T^2}{q^2} - 2 \int_1^\infty \frac{dx_3}{x_3} (1 + (x_3 - 1)^2) \ln x_3 n_B(\sqrt{q^2} x_3/2) \right. \\
& + \int_0^1 \frac{dx_3}{x_3} (1 + (1 - x_3)^2) \ln(1 - x_3) n_B(\sqrt{q^2} x_3/2) \\
& + \int_0^\infty \frac{dx_3}{x_3} (1 + (x_3 + 1)^2) \ln(1 + x_3) n_B(\sqrt{q^2} x_3/2) \\
& \left. + \left( 2 \ln\left(\frac{\theta_m}{2}\right) - \gamma - \ln\left(\frac{q^2}{4\pi\mu^2}\right) \right) \int_1^\infty \frac{dx_3}{x_3} (1 + (x_3 - 1)^2) n_B(\sqrt{q^2} x_3/2) \right].
\end{aligned} \tag{24}$$

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<sup>6</sup>This is a collinear divergent piece which appears due to the absence of Compton scattering processes in the heat bath

We are considering here the hardonic jets arising from the quark-antiquark pair in  $e^+e^-$  annihilation. In order that the hadronic jets follow from the quark-antiquark pair, the quark and antiquark are required not to loose too much energy in their direction by the emission of gluons and quark pairs. In otherwords most of the annihilation energy is deposited along the direction of the quark and antiquark. For the production of 2-jets the three body phase space integration in eqn.(8) for emission process must be restricted. We define the contribution of the real diagrams to the imaginary part of the trace of photon polarisation tensor as

$$Im\Pi_{\mathcal{R}\mu}^{(R)\mu}(q) = \frac{e^2 g^2}{2} C(R) N_c \left( \sum_n Q_n^2 \right) [J_{\mathcal{R}}^{(0)} + J_{\mathcal{R}}^{(em)} + J^{(abs)}] \quad (25)$$

where

$$J_{\mathcal{R}}^{(0)} = \int_{\mathcal{R}} dR_3 X, \quad (26)$$

$$J_{\mathcal{R}}^{(em)} = \int_{\mathcal{R}} dR_3 n_B(K_3) X, \quad (27)$$

$X$  is given by eqn.(9) and the integral region  $\mathcal{R}$  is specified by the following conditions: The emitted gluon is either soft (i.e.  $x_3 \leq \Delta$ ) or collinear to one of the quarks (i.e.  $\theta_{12}, \theta_{23} < 2\delta$ ) where  $x_3 (= 2K_3.q/q^2)$  is the fraction of the energy of the virtual photon carried by the gluon and  $\theta_{13}$  ( $\theta_{23}$ ) is the angle between the gluon and the quark (antiquark). Evaluating the integrals we obtain<sup>7</sup>

$$J_{\mathcal{R}}^{(0)} = -\frac{q^2}{8\pi^3} \left( \frac{q^2}{4\pi\mu^2} \right)^{-\epsilon} \frac{\Gamma(2-\epsilon)}{\Gamma(2-2\epsilon)} \left[ \frac{1}{\epsilon^2} + \frac{1}{\epsilon} \left\{ \frac{3}{2} - \gamma - \ln \left( \frac{q^2}{4\pi\mu^2} \right) \right\} + \frac{1}{2} (2\gamma - 3) \ln \left( \frac{q^2}{4\pi\mu^2} \right) \right. \\ \left. + \frac{1}{2} \ln^2 \left( \frac{q^2}{4\pi\mu^2} \right) - \frac{\gamma}{2} + \frac{\gamma^2}{2} - \frac{11\pi^2}{12} + \frac{13}{2} - 4 \ln \delta \ln \Delta - 3 \ln \delta \right] + 0(\delta, \Delta), \quad (28)$$

$$J_{\mathcal{R}}^{(em)} = \frac{q^2}{8\pi^3} \left( \frac{q^2}{4\pi\mu^2} \right)^{-2\epsilon} \frac{\Gamma(2-\epsilon)}{\Gamma(2-2\epsilon)} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left[ \frac{2}{\epsilon} \int_0^1 dx_3 x_3^{-1-2\epsilon} (1-x_3)^{1-\epsilon} n_B(\sqrt{q^2} x_3/2) \right. \\ \left. + \frac{1}{\epsilon} \frac{1-\epsilon}{1-2\epsilon} \int_0^1 dx_3 x_3^{1-2\epsilon} (1-x_3)^{-\epsilon} n_B(\sqrt{q^2} x_3/2) - \int_0^1 dx_3 x_3 n_B(\sqrt{q^2} x_3/2) \right. \\ \left. - \int_{\Delta}^1 \frac{dx_3}{x_3} n_B(\sqrt{q^2} x_3/2) \left\{ \frac{x_3^2}{1-x_3 \sin^2 \delta} (1 - (2-x_3) \sin^2 \delta) + 2((1-x_3)^2 + 1) \ln \tan \delta \right. \right. \\ \left. \left. + ((1-x_3)^2 + 1) \ln(1-x_3) \right\} \right]. \quad (29)$$

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<sup>7</sup> In addition to  $x_i (= 2q.K_i/q^2$  where  $i = 1, 2, 3$ ) define two more variables [17]  $\xi_l = (1 - \cos \theta_{l3})/2$  ( $l = 1, 2$ ). Among these five variables two are independent and so we choose  $\xi_1$  and  $x_3$  as the independent variables. Region  $\mathcal{R}$  is specified by  $\xi_{max} \leq \xi_1 \leq \xi_{min}$  and  $0 \leq x_3 \leq \Delta$ , where  $\xi_{max} = \frac{1-\sin^2 \delta}{1-x_3(2-x_3)\sin^2 \delta}$  and  $\xi_{min} = \sin^2 \delta$ . Now  $J_{\mathcal{R}}^{(0)} = J^{(0)} - J_{\bar{\mathcal{R}}}^{(0)}$ , where  $\bar{\mathcal{R}}$  is obtained by eliminating  $\mathcal{R}$  from the whole phase space. So  $J_{\bar{\mathcal{R}}}^{(0)} = \frac{q^2}{16(2\pi)^3} \int_{\Delta}^1 dx_3 x_3 (1-x_3) \int_{\xi_{min}}^{\xi_{max}} d\xi_1 \frac{X}{(1-x_3\xi_1)^2}$  where  $X$  is written in terms of  $x_3$  and  $\xi_1$ . Evaluation of  $J_{\mathcal{R}}^{(em)}$  is also carried out in the same fashion.

The 2-jets cross-section for the processes  $e^+e^- \rightarrow \gamma^* \rightarrow q\bar{q}g$  and  $e^+e^- \rightarrow \gamma^*g \rightarrow q\bar{q}$  to  $\alpha_s$  order reads

$$\begin{aligned}\sigma_{2jet} &= -\frac{e^2}{q^4(3-2\epsilon)}[Im\Pi_{\mu}^{(B)\mu}(q) + Im\Pi_{\mathcal{R}\mu}^{(R)\mu}(q) + Im\Pi_{\mu}^{(Ve)\mu}(q) + Im\Pi_{\mu}^{(Se)\mu}(q)] \\ &= \sigma_{2jet}^{(0)} + \sigma_{2jet}^{(T)},\end{aligned}\tag{30}$$

where  $\sigma_{2jet}^{(0)}$  and  $\sigma_{2jet}^{(T)}$  are the zero and finite temperature piece respectively. The expression for them are the following:

$$\sigma_{2jet}^{(0)} = \sigma_B \left[ 1 + \frac{\alpha_s}{\pi} C(R) \left\{ \gamma - \frac{5\pi^2}{6} + \frac{5}{2} - 4 \ln \delta \ln \Delta - 3 \ln \delta \right\} \right],\tag{31}$$

$$\sigma_{2jet}^{(T)} = \sigma_B \frac{\alpha_s}{\pi} C(R) \left[ \frac{4}{3} \pi^2 \frac{T^2}{q^2} + f_2(\sqrt{q^2}/T, \Delta, \delta) \right],\tag{32}$$

where

$$\begin{aligned}f_2(\sqrt{q^2}/T, \Delta, \delta) &= -2 \int_1^\infty \frac{dx_3}{x_3} n_B(\sqrt{q^2}x_3/2) ((x_3-1)^2 + 1) \ln x_3 \\ &\quad + \int_0^1 \frac{dx_3}{x_3} n_B(\sqrt{q^2}x_3/2) (1 + (1-x_3)^2) \ln(1-x_3) \\ &\quad + \int_0^\infty \frac{dx_3}{x_3} n_B(\sqrt{q^2}x_3/2) (1 + (x_3+1)^2) \ln(1+x_3) \\ &\quad + \left( 2 \ln \delta - \gamma - \ln\left(\frac{q^2}{4\pi\mu^2}\right) \right) \int_1^\infty \frac{dx_3}{x_3} n_B(\sqrt{q^2}x_3/2) ((x_3-1)^2 + 1) \\ &\quad + \int_\Delta^1 \frac{dx_3}{x_3} n_B(\sqrt{q^2}x_3/2) \left\{ x_3^2 + 2((1-x_3)^2 + 1) \ln \delta \right. \\ &\quad \left. + ((1-x_3)^2 + 1) \ln(1-x_3) \right\} + 0(\delta, \Delta),\end{aligned}\tag{33}$$

and  $\sigma_B$  is given by the eqn.(18). Here without any loss of generality we have chosen  $\theta_m = 2\delta$ . In real life we can relate  $\delta$  to the maximum resolving power of the detector. According to eqn.(31) and eqn.(32) we see that the order  $\alpha_s$  correction to the two jets from the quark pair is controllable in size within the frame work of perturbation theory at finite temperature. Since the virtual photon is hard, the ratio  $\frac{\sqrt{q^2}}{T}$  is very large. The function  $f_2(\sqrt{q^2}/T)$  is well behaved for large  $\frac{\sqrt{q^2}}{T}$  ratio and it vanishes for  $\frac{\sqrt{q^2}}{T} \rightarrow \infty$ .

3-jets cross-section is the complementary of the 2-jets cross-section. For 3-jets the imaginary part of the trace of photon polarisation tensor reads

$$Im\Pi_{\mathcal{R}\mu}^{(R)\mu}(q) = \frac{e^2 g^2}{2} C(R) N_c \left( \sum_n Q_n^2 \right) [J_{\mathcal{R}}^{(0)} + J_{\mathcal{R}}^{(em)}].\tag{34}$$



The 3-jets cross-section for the process  $e^+e^- \rightarrow \gamma^* \rightarrow q\bar{q}g$  to  $\alpha_s$  order reads

$$\begin{aligned}\sigma_{3jet} &= -\frac{e^2}{q^4(3-2\epsilon)} \text{Im} \Pi_{\bar{\kappa}\mu}^{(R)\mu}(q) \\ &= \sigma_{3jet}^{(0)} + \sigma_{3jet}^{(T)},\end{aligned}\tag{35}$$

where  $\sigma_{3jet}^{(0)}$  and  $\sigma_{3jet}^{(T)}$  are the zero and finite temperature piece respectively. The expression for them are

$$\sigma_{3jet}^{(0)} = \sigma_B \frac{\alpha_s}{\pi} C(R) \left[ 4 \ln \delta \ln \Delta + 3 \ln \delta + \frac{\pi^2}{3} - \frac{7}{3} \right] + 0(\delta, \Delta),\tag{36}$$

$$\sigma_{3jet}^{(T)} = \sigma_B \frac{\alpha_s}{\pi} C(R) f_3(\sqrt{q^2}/T) + 0(\delta, \Delta),\tag{37}$$

where

$$f_3(\sqrt{q^2}/T) = - \int_{\Delta}^1 \frac{dx_3}{x_3} n_B(\sqrt{q^2}x_3/2) \{ x_3^2 + 2((1-x_3)^2 + 1) \ln \delta + ((1-x_3)^2 + 1) \ln(1-x_3) \}.\tag{38}$$

Here  $f_3(\sqrt{q^2}/T)$  is also well behaved for large  $\frac{\sqrt{q^2}}{T}$  ratio and it vanishes for  $\frac{\sqrt{q^2}}{T} \rightarrow \infty$ . Because of  $\ln \delta$  and  $\ln(1-x_3)$  in the second and third term within the second bracket of eqn.(38) respectively we obtain positive contribution after integrations. However the first term within the bracket will give negative contribution. We write the 3-jets cross-section as

$$\sigma_{3jets} = \sigma_B \frac{\alpha_s}{\pi} C(R) h(\sqrt{q^2}/T, \delta, \Delta) + 0(\delta, \Delta),\tag{39}$$

where

$$h = 4 \ln \delta \ln \Delta + 3 \ln \delta + \frac{\pi^2}{3} - \frac{7}{3} + f_3(\sqrt{q^2}/T).\tag{40}$$

The plots of  $h$  versus  $\delta$  (Fig.5 and Fig.6) for different set of values of  $\sqrt{q^2}/T$  and  $\Delta$  show that the function  $h$  remains positive for realistic values of its argument. The maximum value of  $\Delta$  we have taken is 0.4, which is much away from the soft gluon approximation, yet  $h$  remains positive for this large value of  $\Delta$ .

We obtain from eqn.(30) and eqn.(35) that the ratio of the two cross-section is

$$\frac{\sigma_{3jet}}{\sigma_{2jet}} = \frac{\alpha_s}{\pi} C(R) h(\sqrt{q^2}/T, \delta, \Delta) + 0(\alpha_s^2).\tag{41}$$

Apart from the small numerical factor  $\frac{\alpha_s}{\pi} C(R)$  this ratio behaves in the same way as the function  $h$ . As  $\delta$  and  $\Delta$  decrease the 3-jets phase space gets increased compared to the 2-jets, as a result this ratio gets increased which are evident from the plots in Fig.5 and Fig.6. It is also evident from these

plots that for a fixed value of  $\delta$  and  $\Delta$  as  $\sqrt{q^2}/T$  increases the phase space of gluons get decreased due to Bose-Einstein distribution function and as a result this ratio gets decreased.

I gratefully acknowledge S. Gupta for suggesting the problem and for the many subsequent discussions and helpful comments. I also thank P. Roy and K. Sridhar for helpful discussions and suggestions. I am also thankful to P. Aurenche for clarifying certain points regarding the infrared cancellations in their paper [3] and also for helpful comments.

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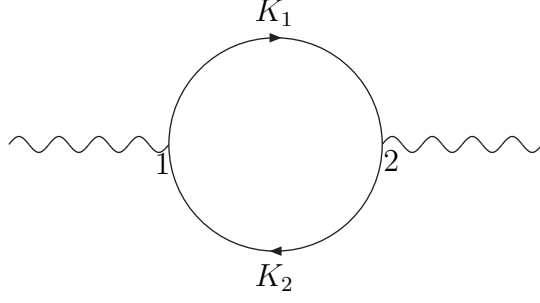


Figure 1: One loop photon polarization graph for the computation of Born amplitude.

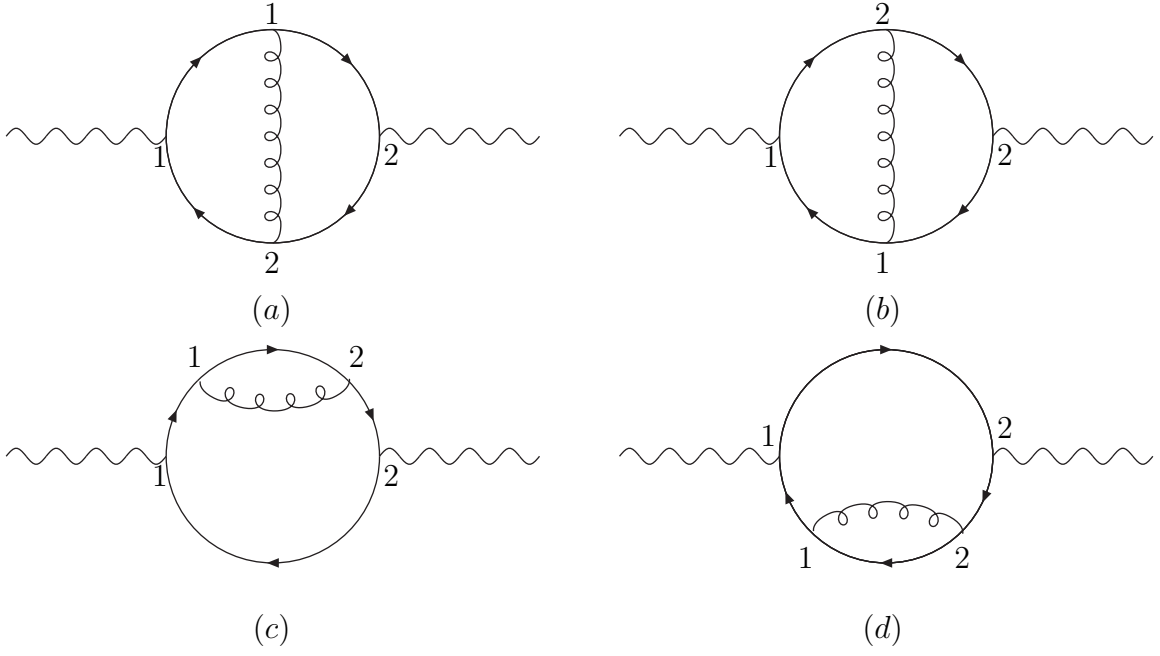


Figure 2: Two loop photon polarization graph for the computation of  $\gamma^* \rightarrow q\bar{q}g$  and  $\gamma^*g \rightarrow q\bar{q}$  amplitude .

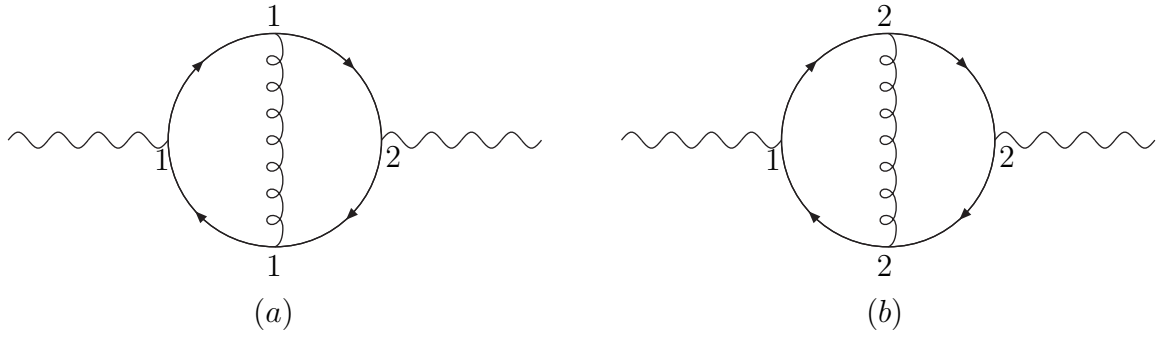


Figure 3: Two loop photon polarization graph for the computation of vertex correction of  $\gamma^* \rightarrow q\bar{q}$  amplitude to  $\alpha_s$  order.

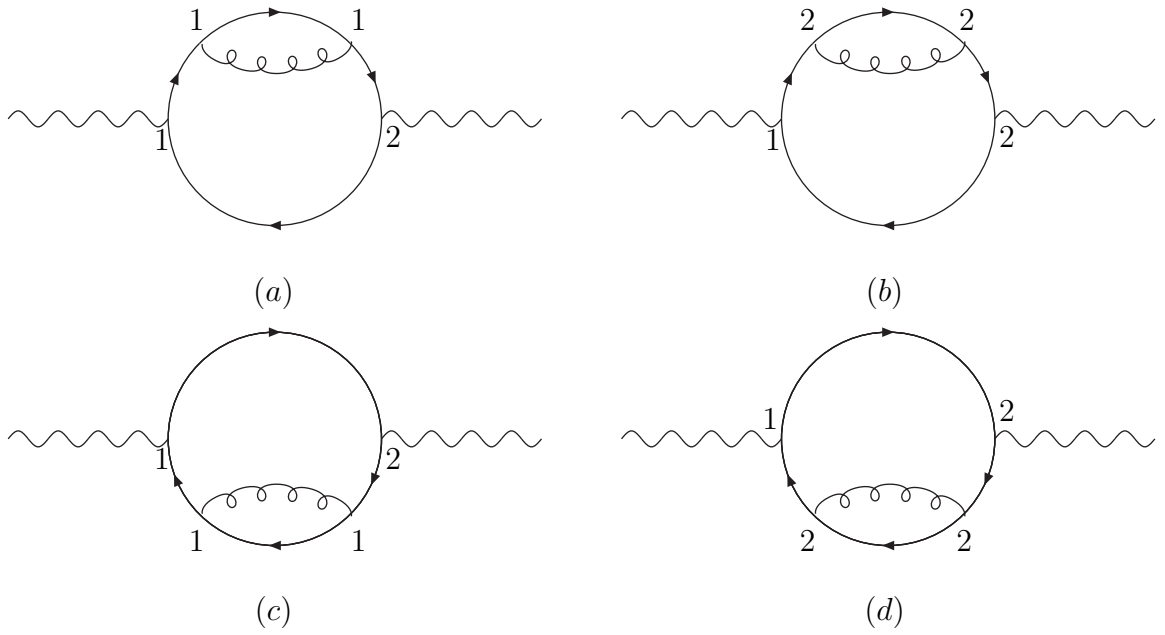


Figure 4: Two loop photon polarization graph for the computation of self energy corrections of  $\gamma^* \rightarrow q\bar{q}$  amplitude to  $\alpha_s$  order.

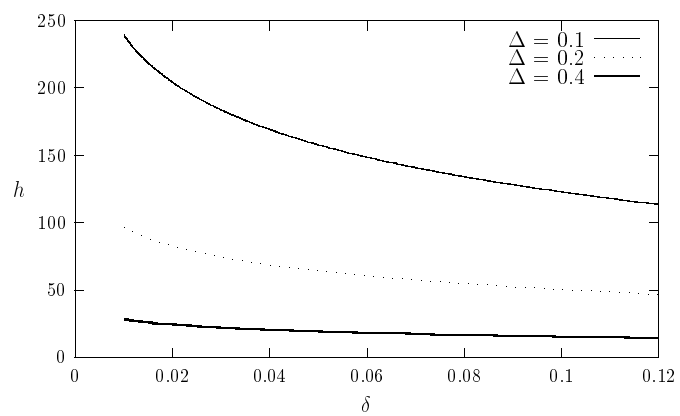


Figure 5: Plot of  $h$  versus  $\delta$  for different values of  $\Delta$  and  $\sqrt{q^2}/T = 1.2$ .

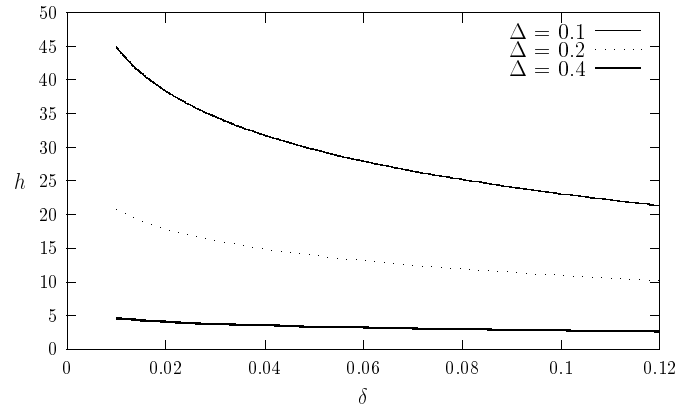


Figure 6: Plot of  $h$  versus  $\delta$  for different values of  $\Delta$  and  $\sqrt{q^2}/T = 10$ .